

# On a Generating Function and its Probability Distributions. A Contribution to the Theory of Transition Rates I

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A function of two complex variables with two real parameters  $a$  and  $b$  is described, which generates a sequence of probability distributions of two integer variables  $m \geq 0$  and  $n \geq 0$ . Closed expressions for the special  $b = 0$  and general case  $b \neq 0$  and recurrence equations for calculating the probability distributions are derived. The probability distribution for  $m = 0$  and  $a$  large enough is qualitatively bell-shaped, and that for  $m \neq 0$  has multiple peak structures. In both cases, the  $b$  parameter influences solely the skewness of the curves. For small  $a$  values, the distributions fall rapidly from a value of nearly one, decreasing by a factor of  $10^{10}$  or more as  $n$  increases from zero to  $n = 10$ . The influence of the  $b$  parameter on their properties can be pronounced. Finally, we note an important property of the distributions when two or several of them are convoluted with one another. The result is expressed in terms of an addition theorem in respect to the parameter  $a$  and describes a multidimensional distribution.

*Key words:* Generating Function; Function of Two Complex Variables; Probability Distributions; Multidimensional Probability Distributions; Transition Probabilities.